

Implementation of MEWMA Control Chart in Equipment Condition Monitoring

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Abstract

The progressive degradation of presently operating electro-mechanical systems is a certain future fact. To minimize losses, maintenance costs and eventual replacements, condition monitoring should be applied to critical equipment (Condition Based Maintenance – CBM). The state of equipment can be predicted at any moment using statistical methods to analyze condition monitoring data. In this paper, collected data are vibration values, obtained at p points ($p = 4$ for instance) of an experimental equipment, forming p variables. When independence condition does not hold, it is suggested modeling data with Auto-Regressive Integrated Moving Average (ARIMA) models, and using the residues of the estimated model for Phase I. In Phase I, the estimation of parameters is achieved using the Hotelling T^2 control chart; only after applying the defined ARIMA model, the p variables are treated. In Phase II, equipment state is artificially degraded through induced failures and failure prediction obtained using special multivariate control charts for data statistical treatment. Assuming data independence and normality, Multivariate Exponentially Weighted Moving Average Modified (MEWMAM) control charts are applied in Phase II to data collected from an electric pump, controlling the behavior of data using this procedure. In Phase II, for non-independent data the prediction errors from the adjusted model are used instead of original data. To show that the suggested methodology can be applied to propulsion systems, simulated data from a gas turbine are used. Using these methodologies it is possible to run online condition monitoring, and act in time, to minimize maintenance costs and maximize equipment performance.

Keywords: Condition monitoring, Vibration detection and analysis, Statistical process control, Multivariate Exponential Weighted Moving Average (MEWMA) control chart

Acronyms

AL Alert Level
AR Auto-regressive
AR2 Auto-regressive of order 2

ARIMA	Auto-regressive integrated Moving Average
ARMA	Mix of AR and MA
ACF	Auto-correlation Function
ARL	Average Run Length
CBM	Condition Based Maintenance
CL	Center Line
CUSUM	Cumulative Sum
EWMA	Exponentially Weighted Moving Average
EWMAM	Exponentially Weighted Moving Average Modified
EACF	Estimated Auto-correlation Function
EPACF	Estimated Partial Auto-correlation Function
GG	Gas Generator
k	Observation instant for the 4 variables
k	Safety Factor for T_L calculus
K	Upper Control Level in MEWMAM Control Chart
$K1$	Alert Level in MEWMAM Control Chart
kW	kilo Watt
MA	Moving Average
MEWMA	Multivariate Exponentially Weighted Moving Average
MEWMAM	Multivariate Exponentially Weighted Moving Average Modified
PACF	Partial Auto-correlation Function
PT	Power Turbine
SPC	Statistical Process Control
rpm	Rotation Per Minute
T^2	Hottelling Control Chart, or Multivariate Traditional Control Chart
T^2M	Hottelling Modified Control Chart, or Multivariate Traditional Modified Control Chart
$(T_L)_N$	Limit Defined by Normative or the manufacturer
UCL	Upper Control Level

1 Introduction

When implementing a condition based maintenance system, the type and equipment performance must be considered. The main targets are to reduce maintenance costs and optimize operation.

Statistical methods are applied to condition estimation, the main objectives being to categorize anomalies, identify components involved in those anomalies, estimate possible trends and predict the time to failure^[1].

For repairable systems, the data can be collected using fixed sensors, installed in key points, or using portable collectors.

To decide which features to measure, those that best represent the equipment behavior must be chosen. For rotating equipment, vibrations, oil analysis, and thermography are the most representative of its states^[2]; these methods are suggested since no intervention is needed as part of the data collection task. For the present study, vibration measurement was the selected method and a portable vibration collector was used, the uploading of data being controlled using the equipment software. To define the vibration parameters and monitoring, Statistical Process Control (SPC) with some modifications was applied.

2 Control Charts

Given the right conditions, multivariate control charts can be applied for process control. In this work Hotelling T^2 control chart was used for equipment monitoring in Phase I, both to check the equipment stability and to define its parameters; the MEWMAM control chart was used for the monitoring in Phase II.

2.1 Phase I – T^2 traditional control charts

The design of T^2 control charts obey some rules, the first one being that more than one variable should be observed, and then enough data should be collected to estimate the process parameters. Given the data specifications, individual observations ($n = 1$) control charts will be used^[3].

2.1.1 Independent data

If the observed p variables in control are independent, a suitable model is $X_{ij} = \mu_j + \varepsilon_{ij}$, where X_{ij} is the observation i for variable j , μ_j is the process mean for the variable j , ε_{ij} are iid normal random variables with mean zero and standard deviation σ_ε (white noise).

In Phase I, with m individual observations at each point, X_{ij} ($i = 1, \dots, m; j = 1, \dots, p$), the mean, (\bar{X}_j), the variance S_{ij} , and the covariance S_{jh} are calculated for each point and each pair of points. The vector mean (\bar{X}) and the covariance matrix (S) are given by the next expressions^[4]:

$$\bar{X} = (\bar{X}_1, \bar{X}_2, \dots, \bar{X}_p)^T \quad S = \begin{bmatrix} S_{11} & S_{12} & S_{13} & \dots & S_{1p} \\ S_{21} & S_{22} & S_{23} & \dots & S_{2p} \\ \cdot & \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \cdot & \dots & \cdot \\ S_{p1} & S_{p2} & S_{p3} & \dots & S_{pp} \end{bmatrix} \quad (1)$$

For each observation instant k the T^2 control charts are based on the statistic given by (2):

$$(T^2)_k = (X_k - \bar{X})^T S^{-1} (X_k - \bar{X}), \quad (k = 1, 2, \dots, p) \quad (2)$$

The Center Line (CL) and the Upper Control Limit (UCL) for the Phase I are defined in Table 1. The expression $\beta_{\alpha; p/2; (m-p-1)/2}$ is the right percentile, for a probability α , from the beta distribution with the parameters $p/2$, $(m-p-1)/2$. The rule applied to estimate the parameters for T^2 control charts in Phase I, is to have the sample size m ($180p \leq m \leq 300p$)^[4].

Table 1 T^2 control chart limits

Chart	CL	UCL
Phase I	0	$\frac{(m-1)^2}{m} \beta_{\alpha; p/2; (m-p-1)/2}$

2.1.2 Auto-correlated data

If a variable has a significant auto-correlation, T^2 statistic is estimated using the residues from observations; otherwise, the procedure may suggest unnecessary preventive interventions for equipment in good state. The variables X_k , \bar{X} and S are replaced with the corresponding residues and the mean vector and covariance residues matrix are built. For correlated observations, data is modeled with an ARIMA (p, d, q) model^[5] and the corresponding residues are obtained, according with the following procedure. The process auto-correlation is analyzed studying the Auto-Correlation Function (ACF) and the partial auto-correlation function (PACF). To identify a suitable model member from family ARIMA (p, d, q) , the functions (ACF) and (PACF) are compared using the corresponding estimations: the Estimated Auto-Correlation Function (EACF) and the Estimated Partial Auto-Correlation Function (EPACF)^[6,7].

A process follows an ARIMA (p, d, q) model when $\nabla^d X_t$ -the order d difference process-follows an ARMA (Auto-regressive Moving Average) (p, q) model. The model defined by ARIMA (p, d, q) is:

$$\Phi_p(B)\nabla^d = X_t = \Theta_q(B)\varepsilon_t \quad (3)$$

$$\Phi_p(B) = (1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p) \quad (4)$$

$$\Theta_q(B) = (1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q) \quad (5)$$

Here Φ_p and Θ_q are, respectively, symbolic polynomials of order p and q on B . See^[4] and^[7] for details. The Backshift operator B is defined by:

$$B(X_t) = X_{t-1} \quad (6)$$

and the delta operator by:

$$\nabla(X_t) = X_t - X_{t-1} = (1 - B)(X_t) \quad (7)$$

See^[5] and^[7] for details.

The residues corresponding to the specified model are $e_t = X_t - \hat{X}_t$, where \hat{X}_t is an estimation of the process expected value for the period t . The T^2 control chart is built using those residues. The process mean is estimated using (8) below when the process is modeled by an AR(p) (Auto-regressive) = ARIMA($p, 0, 0$) or by a MA(q) (Moving Average) = ARIMA($0, 0, q$)^[7]

$$E(X) = \mu = \frac{\bar{\xi}}{1 - \sum_{j=1}^p \phi_j} \quad E(X) = \mu \quad (8)$$

2.2 Phase II – special control charts – modified MEWMA

Cumulative sum modified control charts (CUMSUM) are chosen instead of traditional control charts, given its sensitivity to small changes.

The EWMA (Exponentially Weighted Moving Average) multivariate modified control chart, MEWMAM, to control the mean is based on the T^2 statistics^[8], defined for instant i , ($i = 1, 2, \dots$) by:

$$T_i^2 = Z_i' \sum_Z^{-1} Z_i \quad (9)$$

where

$$Z_i = RX_i + (I - R)Z_{i-1}, \quad Z_0 = 0, \quad \sum_Z = \frac{\lambda}{2 - \lambda} (1 - (1 - \lambda)^{2i}) \sum \quad (10)$$

In this equation, I is the identity matrix and $R = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_p)$, where $\lambda_j (j = 1, 2, \dots, p)$ is a weighting constant for variable X_j . Usually $\lambda_1 = \lambda_2 = \dots = \lambda_p = \lambda$. In this case, Z_i and the inverse matrix, are defined respectively by:

$$Z_i = \lambda X_i + (1 - \lambda)Z_{i-1} \quad \sum_{-1}^{-1} (Z_i = \lambda X_i + (1 - \lambda)Z_{i-1}) \quad (11)$$

The modification of MEWMA statistics is based, for each variable j , on the maximum acceptable vibration level T_L . This value is calculated as a function of the maximum acceptable vibration level given by a norm or standard $(T_L)_{\text{Standard}}$, the standard deviation σ and a safety factor k^1 , as follows:

$$(T_L)_j = ((T_L)_{\text{Standard}} - k\sigma)_j \quad (12)$$

T_L is used instead of the mean; the use of the mean is correct for industrial production, but, for equipment monitoring it could lead to unnecessary interventions. Since data is auto-correlated and we are applying the MEWMA, the expressions (13) for Z_i and the inverse matrix will be used. The e_i is the predicted error calculated based on the ARIMA model

$$Z_i = \lambda(e_i - T_L) + (1 - \lambda)Z_{i-1} \quad \sum_{-1}^{-1} (\lambda(e_i - T_L) + (1 - \lambda)Z_{i-1}) \quad (13)$$

An out of control situation is detected when $T_i^2 > H$, where H is the control limit.

In^[8] the values of H for $ARL_{\text{In Control}} = 200$ and for $\delta(\mu)$ are suggested when:

$$p = 2, 4, 6, 10, 15(0, 5; 1, 0; 1, 5; 2, 0; 3, 0) \quad \text{and} \\ \lambda = (0, 05; 0, 10; 0, 20; 0, 30; 0, 40; 0, 50; 0, 60; 0, 80).$$

From the same reference optimal values for MEWMA control chart can be obtained, the best values λ and H for $p = 4, 10, 20$ are for $ARL_{\text{In Control}} = 500$, $ARL_{\text{In Control}} = 1000$, for different values of $\delta(\mu)(0, 5; 1, 0; 2, 0; 3, 0)^{[9]}$.

MEWMAM control chart, like univariate EWMA control chart, shows high sensitivity when compared with T^2 and χ^2 , considering small and moderate shifts^[8].

3 Methodology

To define the methodology for monitoring data, sample size (m) is a fundamental input for specified statistical procedures.

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¹Assuming normal distributed data and after some empirical experimentation with confidence interval, we suggest $k = 3$, because this value seem more consistent with the state of the experimental equipment.

2. In Phase I, data is checked for independence, comparing the EACF and the EPACF. If data is auto-correlated, the ARIMA model should be applied, and the T^2 control chart is calculated for the residues; if not, the original data are used. For normal and stable data distribution, the mean vector and the covariance matrix are calculated. For this Phase, at least 200 samples of individual observations must be taken.
3. In the Phase II the MEWMAM control chart is used to monitor the equipment. In this Phase the equipment is monitored since the first observation, and only under defined rules (see below) preventive actions are taken. To define those rules, the value $(T_L)_{\text{Standard}}$ described above should be evaluated according to international standards or manufacture's rules. To define the control chart limits:
 - Estimate the two limits to control the mean level of vibration, specifically, the Upper Control Limit and the Alert Value (AL).
 - Based on ISO 2372:2003 Standard specify the vibration level at which the system must suffer an intervention.
 - Rules to act on the system:
 - Execute an intervention to detect any anomalous situation when 8 consecutive points are above the AL.
 - Proceed to a maintenance intervention when 5 consecutive points above UCL are observed.

4 Case Study

4.1 Real data prototype

For the case study, vibration data from an electro-pump were used. Four points representing the machine state were selected for the data collection.

To test the MEWMA sensitivity, an anomaly with four state aggravations was forced. In what follows vibration values and Root Mean Square (RMS) are expressed in velocity units (mm/s).

4.1.1 Phase I – T^2 control chart

Assuming the electro-pump working properly, for Phase I, the equipment parameters are estimated based on 241 individual observations. The first step is testing the data for independence, comparing ACF with PACF using its estimators EACF and EPACF for the four variables, the result being a model that best fits the data. In the present case, the result is an auto-correlated data set, so we use the residues from the adjusted model in the Shewhart control charts. In this specific case, the residues are studied using the Statistica software. Considering only, for illustrative purposes, the variable Var 2, a point above the UCL was detected, so it was replaced using the defined model. Figure 1 shows the EACF and EPACF for the variable 2, where auto-correlation is significant and we can see a AR2 (Auto-regressive of order 2) ARIMA model, where for ACF we have an exponential decrease after a certain lag order, and for PACF significant peaks through lags $\log(p)$ (lag 1 and lag 2).

All the four variables are well fitted with an AR2 ARIMA model; given space limitations we show only values corresponding to Var 2: $\xi = 0, 1025$, $\varphi_1 = 0.4945$ and $\varphi_2 = 0.2919$.

Although not shown here, all variables have been studied and follow a normal distribution. When we analyze the T^2 control chart, no special causes of variation are detected, so the mean vector

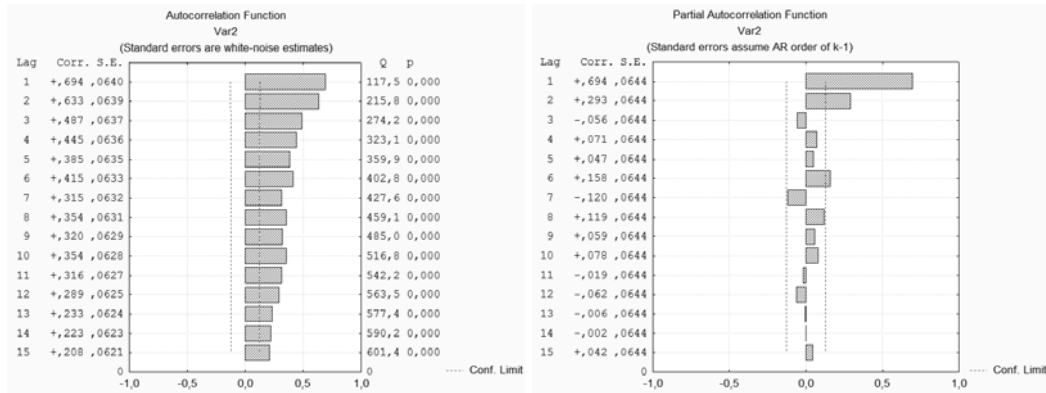
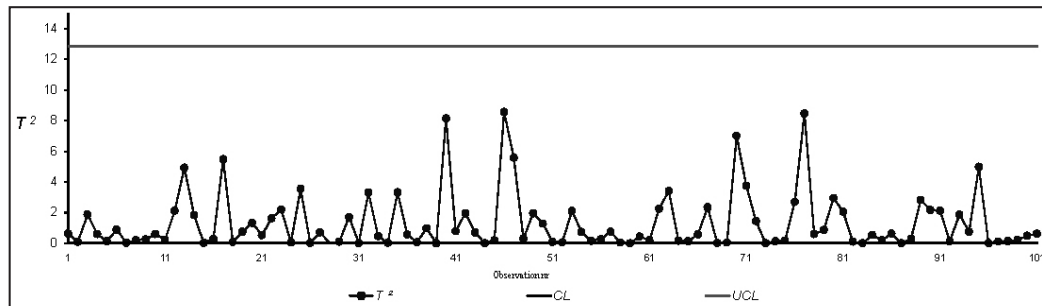


Fig. 1 EACF and EPACF for Var 2

Fig. 2 T^2 control chart – Phase I

(residues and variables) and covariance matrix are estimated^[3]. For graphic reasons, in Fig. 2, only 100 of the 241 samples are shown

$$\bar{e} = \begin{bmatrix} 0,00033 \\ 0,00021 \\ 0,00034 \\ 0,00041 \end{bmatrix} \quad \bar{X} = \begin{bmatrix} 0,406 \\ 0,4810 \\ 0,7400 \\ 0,5220 \end{bmatrix}$$

$$S = \begin{bmatrix} 0,000726 & -0,000057 & 0,000137 & 0,000035 \\ -0,000057 & 0,000686 & -0,000123 & -0,000007 \\ 0,000137 & -0,000123 & 0,007102 & 0,000298 \\ 0,000035 & -0,000007 & 0,000298 & 0,001627 \end{bmatrix}$$

4.1.2 Phase II – MEWMAM control chart

For Phase II the limit vibration was extracted from the ISO 2372:2003. For an electrical engine with 1.5 kW, 1.12 mm/s (RMS) is the allowable limit of an equipment of class I; let this value be named as $(T_L)_N = 1.12$.

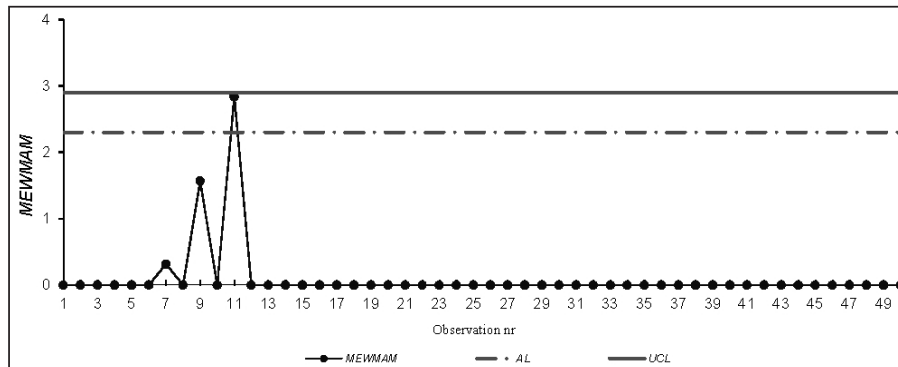


Fig. 3 3ª Aggravation MEWMAM control chart

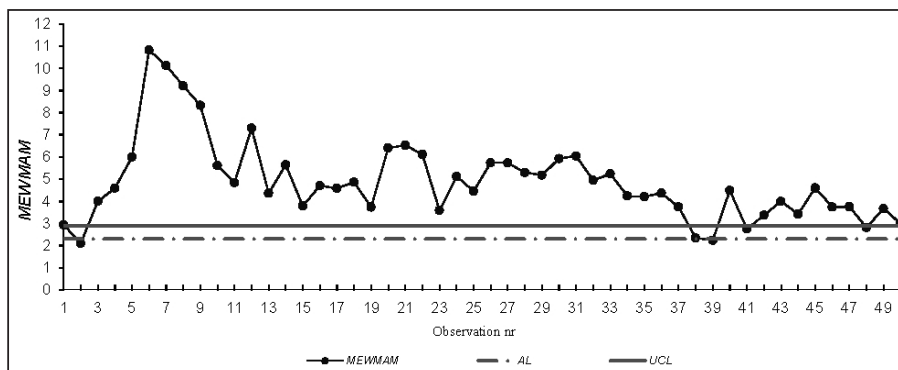


Fig. 4 4ª Aggravation MEWMAM control chart

Having $p = 4$, according Crowder (1989) abacus, the limits for MEWMAM control charts are presented in Table 1, so the K_1 is the Alert Level (2,1) and K_2 (2,7) the Upper Control Limit.

The vibrations are monitored since the first data value. To test the control chart sensitivity, 50 individual observations were read with the electric pump in normal operation; then, to accelerate the degradation, an anomaly and its aggravation through 4 stages was introduced.

Building the MEWMAM control chart to monitor the equipment, until the second aggravation no point is shown in the control charts.

For the third aggravation 3 points appear, all of them are under the UCL (Fig. 3).

The use of MEWMAM control chart shows a high level of sensitivity because after the third observation almost all of the data are above the upper control limit, Fig. 4.

4.2 Data simulation – gas turbine

The equipment under study is a split-shaft Gas Turbine with a thermal efficiency of 37%, 19575 kW and a power velocity of 3600 rpm.

The developed methodology monitors the gas turbine condition, which is a part of the ship propulsion system.

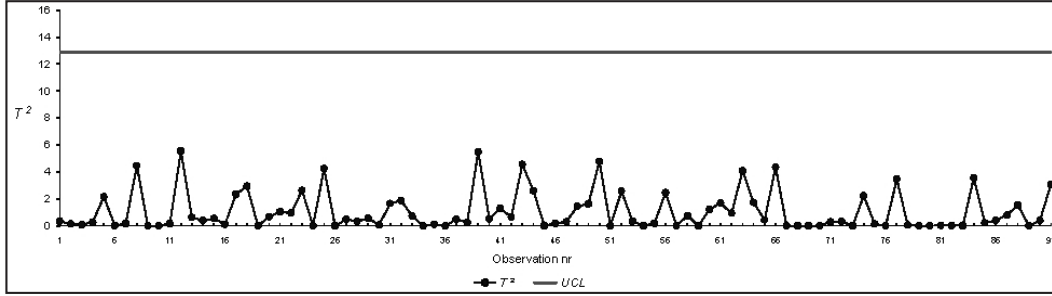


Fig. 5 Phase I – T^2 control chart

The LM2500 Gas Turbine has two parts, the Gas Generator (GG) and the Power Turbine (PT). The compressor, combustion chamber and the high pressure Turbine compose the GG.

Four sensors are considered in this study: the self and induced gas generator vibration sensors and the self and induced power turbine vibration sensors.

To illustrate the methodology application to propulsion equipment, based on collected real data from the gas turbine, 200 bootstrap samples were generated out of the observed data, using the resampling MATLAB function *bootstrap*^[10] to define the equipment parameters on the Phase I. On Phase II the mean and standard deviation were varied to simulate anomalies and to test the control chart sensitiveness to this kind of data.

For each measured point the vibration amplitude unit is expressed in mm, and it represents the vibration global value. Before the data simulation, the gas turbine was assumed to be working for 30 minutes.

The manufacturer characteristics define the vibration limits. For the gas generator at 4 mils there is an alert, and at 7 mils the gas turbine stops. For the power turbine at 7 mils there is a warning, and to 10 mils of vibration the turbine there is an emergency stop.

The generated data using the procedure *bootstrap* from MATLAB was independent and normal^[10]. Thus in the study we start to define the equipment parameters by applying the T^2 control chart for the Phase I.

4.2.1 Phase I – T^2 control chart

In the application of T^2 for simulated data the control limits are obtained from Table 1. See Fig. 5.

All observations are under the upper control limit, thus control charts parameters can be estimated. The mean vector and the covariance inverse matrix are given by:

$$\bar{X} = \begin{bmatrix} 93, 11 \\ 93, 34 \\ 90, 63 \\ 73, 88 \end{bmatrix} \quad S = \begin{bmatrix} 0, 27225 & 0, 15876 & 0, 08245 & 0, 20297 \\ 0, 15828 & 0, 51654 & 0, 07243 & 0, 13266 \\ 0, 08245 & 0, 07243 & 0, 25392 & 0, 31507 \\ 0, 20297 & 0, 13266 & 0, 31507 & 0, 73457 \end{bmatrix}$$

$$S^{-1} = \begin{bmatrix} 5, 4 & -1, 343 & 0, 385 & -1, 412 \\ -1, 3429 & 2, 4 & -0, 3624 & 0, 0979 \\ 0, 3847 & -0, 3624 & 8, 48 & -3, 678 \\ -1, 412 & 0, 09792 & -3, 678 & 3, 31 \end{bmatrix}$$

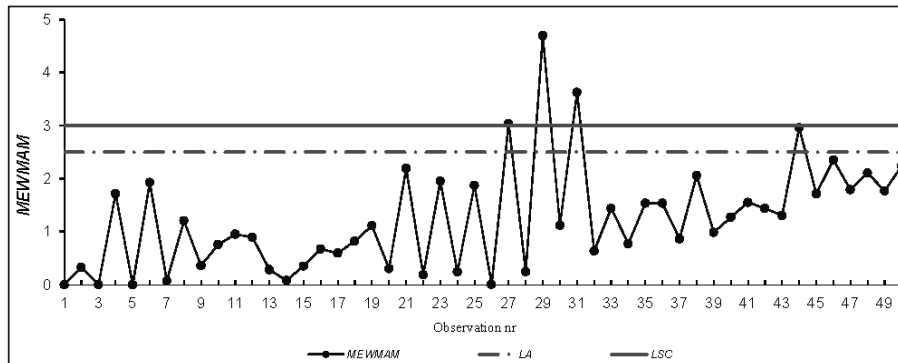


Fig. 6 Phase II – MEWMAM – 3rd anomaly aggravation

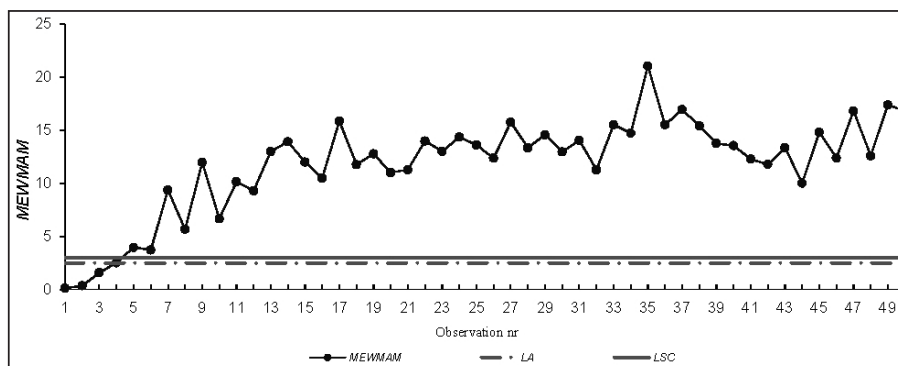


Fig. 7 Phase II – MEWMAM – 4th anomaly aggravation

4.2.2 Phase II – MEWMAM control chart

As for the electro-pump, the data were monitored since the first observation. The values were generated varying the mean and the standard deviation, considering $n = 1$, and 50 observations were generated for each situation of aggravation. The considered limits were:

$$T_L = \begin{bmatrix} 100,035 \\ 176,606 \\ 100,739 \\ 176,448 \end{bmatrix}$$

In this data simulation for gas turbine, $p = 4$, and the Crowder (1989) abacus for the MEWMAM control chart limits supplied an alert level of $K_1 = 2.5$, and an upper control limit of $K = 3$.

In Fig. 6, for the 3rd anomaly aggravation, three non-consecutive observations above the UCL can be noticed. Applying the rules, there is no need for a preventive intervention.

For the 4th anomaly aggravation (Fig. 7), after the 9th observation, the need for a maintenance intervention shows up. All observations after the 5th are above the UCL.

5 Conclusions

The Statistical Process Control was modified to account for the objective of equipment monitoring.

To monitor equipment working condition, the limits should be defined by normative or by the manufacturer.

For the prototype, in Phase I, auto-correlation was detected in data. According to rules, T^2 control charts were applied using the residues from a fitted ARIMA model. On the Phase II, the MEWMA was applied to the expected values; high sensitivity was shown only after the 4th anomaly aggravation.

For the gas turbine, the application of T^2M control chart (T^2 modified control chart) is also possible for the Phase I, using simulated data. In Phase II, the gas turbine monitoring in working condition is possible using the MEWMAM control chart based on the calculated parameters.

Results from the prototype and from the gas turbine support the fact that equipment vibration monitoring is possible using the developed methodology for the Phase I and Phase II.

In spite of the results from Phase II for the referred equipment, it is considered that more tests, and maybe application to other equipment, should be carried out.

The application of a special multivariate control chart allows fast detection of eventual anomalies.

The developed methodology will contribute to avoid unexpected equipment failures, increasing the operational reliability and the availability.

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